Optimal Design for A/B Testing in Two-sided Marketplaces

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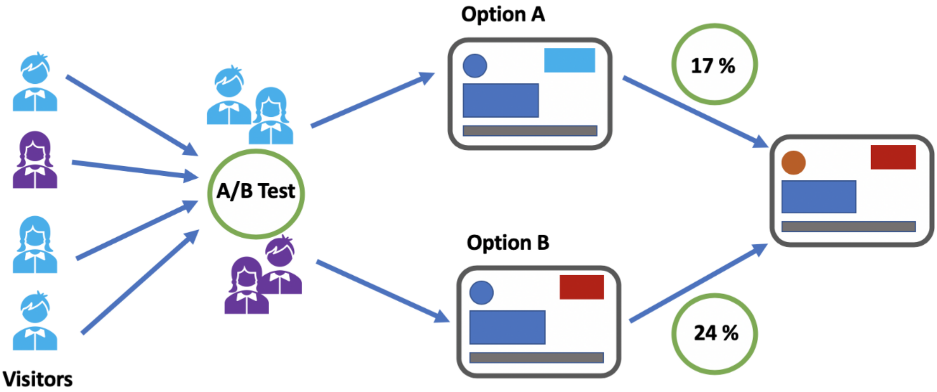
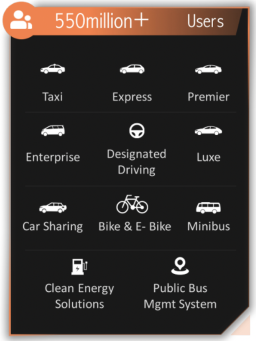
A/B Testing



Taken from
`https://towardsdatascience.com/how-to-conduct-a-b-testing-3076074a8458`

Ridesharing



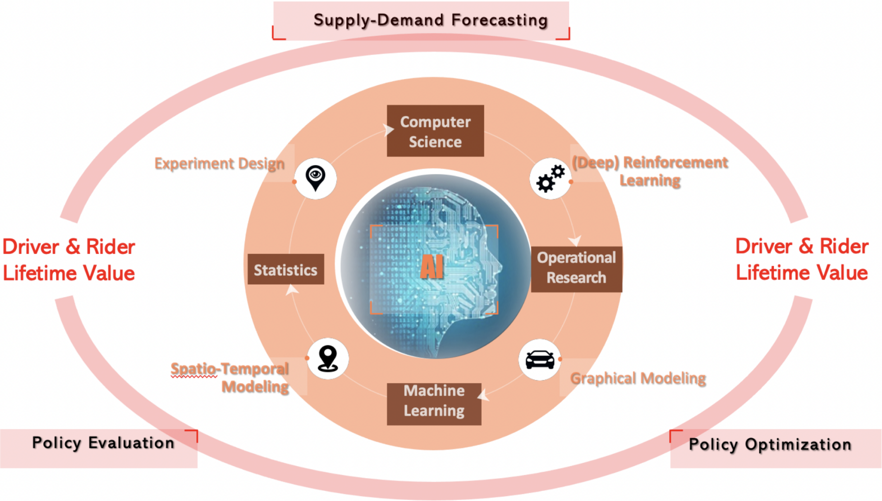
106TB+ vehicle trajectory data/day

4875TB+ data processed/day

40billion+ routing requests/day

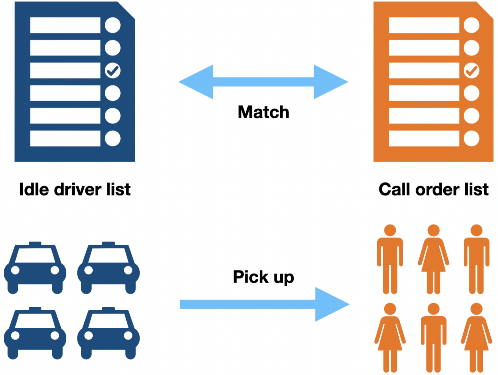
15billion+ location points/day

Ridesharing (Cont'd)



Policies of Interest

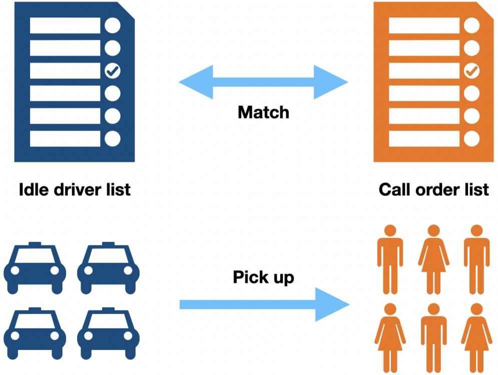
- **Order dispatching**



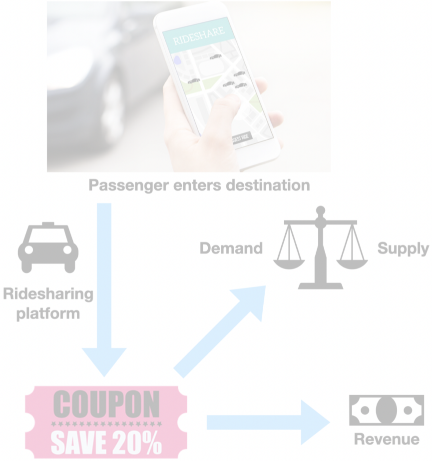
- **Subsidizing**

Policies of Interest

- **Order dispatching**



- **Subsidizing**



Time Series Data

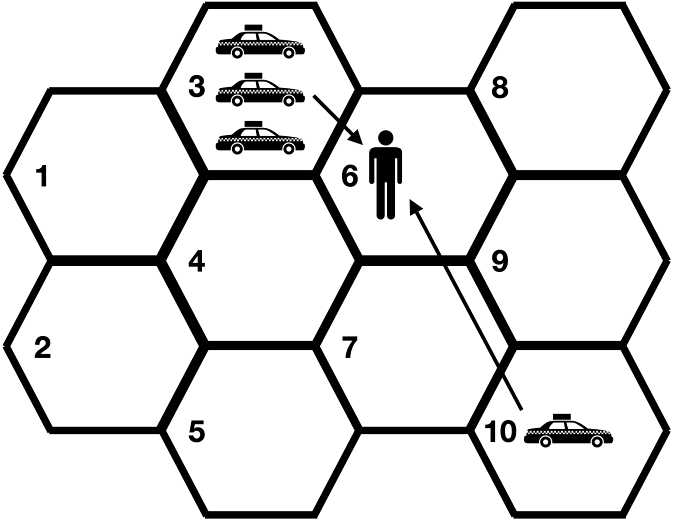
- Online experiment typically lasts for **two weeks**
- **30 minutes/1 hour** as one time unit
- Data forms a **time series** $\{(Y_t, U_t) : 1 \leq t \leq T\}$
- **Observations** $Y_t \in \mathbb{R}^3$:
  1. **Outcome**: drivers' income or no. of completed orders
  2. **Supply**: no. of idle drivers
  3. **Demand**: no. of call orders
- **Treatment** $U_t \in \{1, -1\}$:
  - **New** order dispatching policy B
  - **Old** order dispatching policy A

Challenges

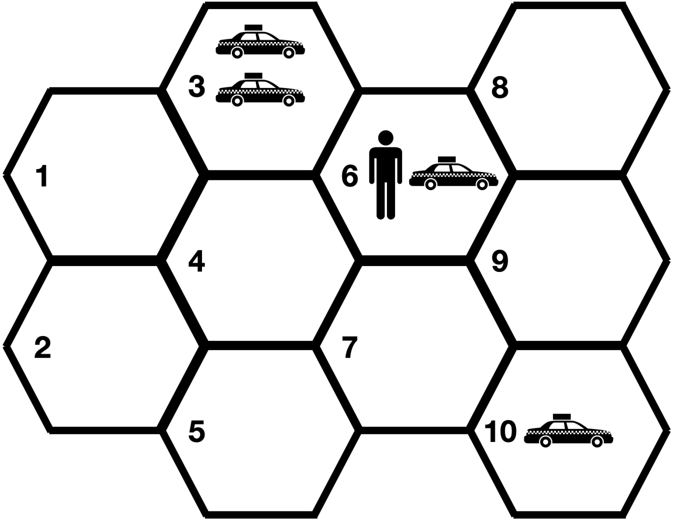
1. **Carryover Effects**:
   - Past treatments influence future observations [Li et al., 2024, Figure 2] $\longrightarrow$
   - Invalidating many conventional A/B testing/causal inference methods [Shi et al., 2023].
2. **Partial Observability**:
   - The environmental state is not fully observable $\longrightarrow$
   - Leading to the violation of the Markov assumption.
3. **Small Sample Size**:
   - Online experiments typically last only two weeks [Xu et al., 2018] $\longrightarrow$
   - Increasing the variability of the average treatment effect (ATE) estimator.
4. **Small Signal**:
   - Size of treatment effects ranges from 0.5% to 2% [Tang et al., 2019] $\longrightarrow$
   - Making it challenging to distinguish between new and old policies.

To our knowledge, **no** existing method has simultaneously addressed all four challenges.

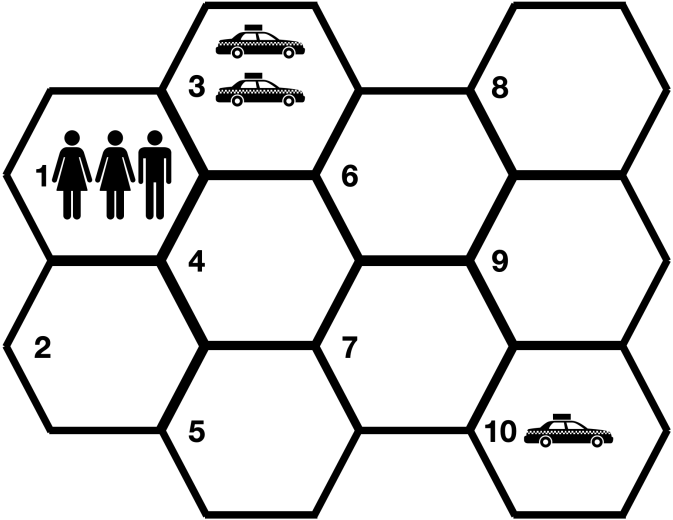
Challenge I: Carryover Effects



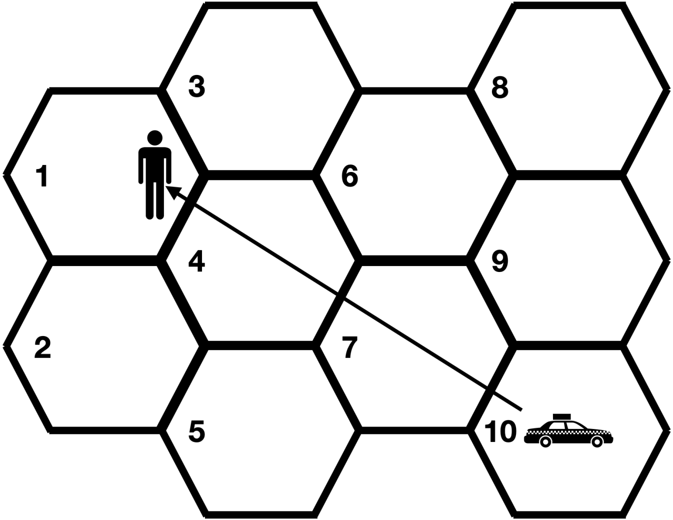
Adopting the Closest Driver Policy



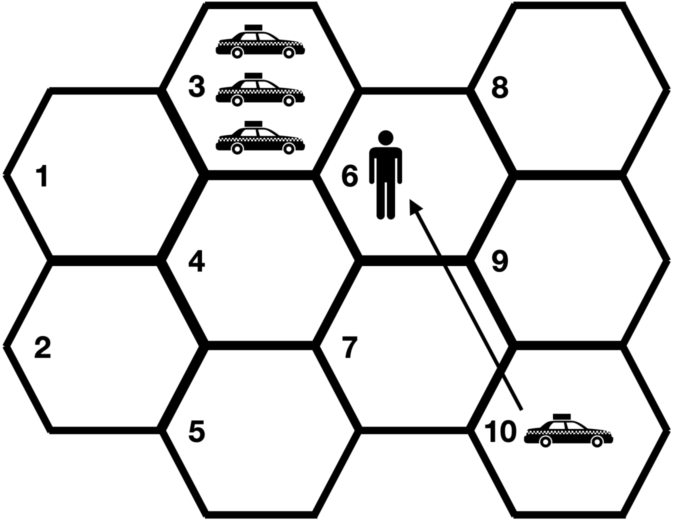
Some Time Later ...



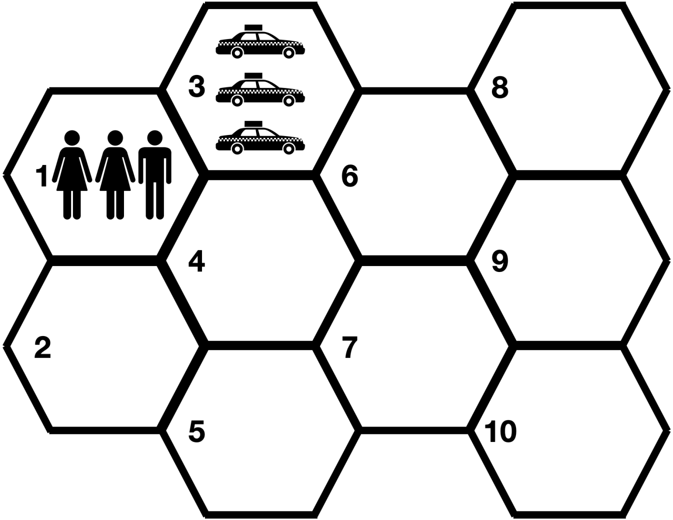
Miss One Order



Consider a Different Action



Able to Match All Orders

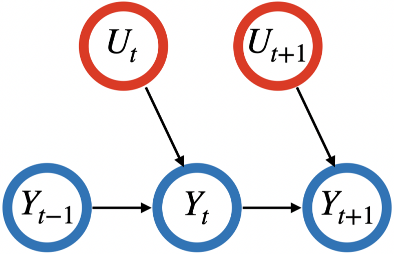


Challenge I: Carryover Effects (Cont'd)

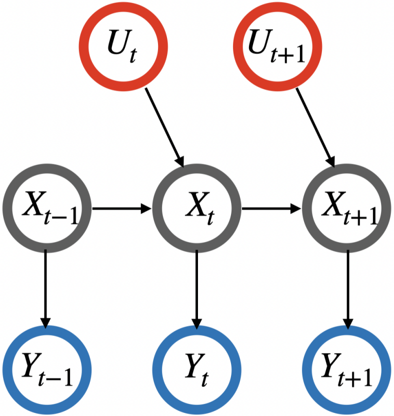
past treatments $\rightarrow$ distribution of drivers $\rightarrow$ future outcomes

Challenge II: Partial Observability

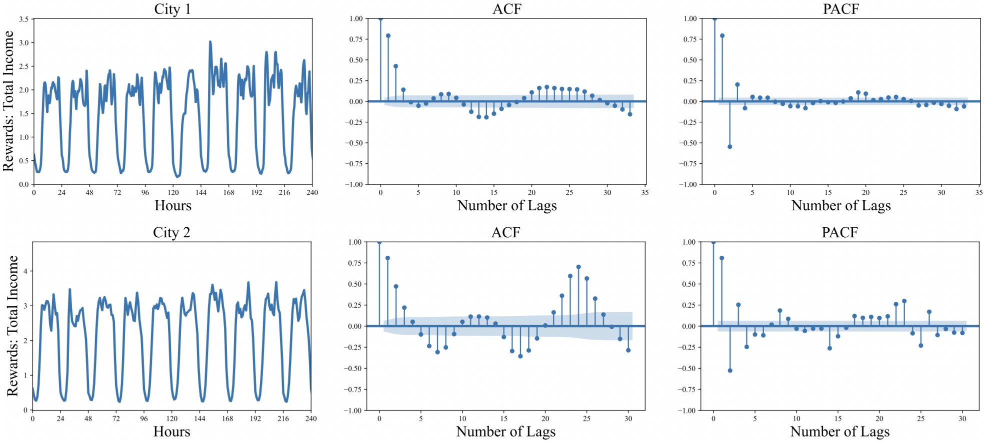
- **Fully Observable Markovian Environments**



- **Partially Observable non-Markovian Environments**



Challenge II: Partial Observability (Cont'd)



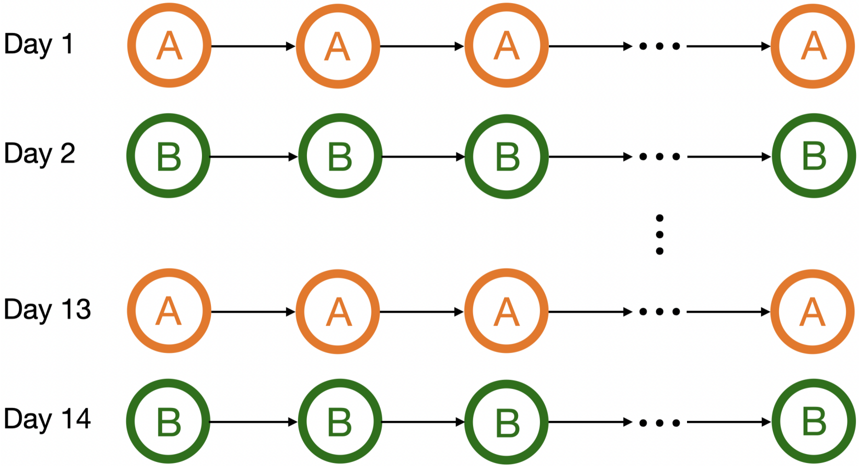
Average Treatment Effect

- Data summarized into a **time series** $\{(Y_t, U_t) : 1 \leq t \leq T\}$
- The first element of Y_t – denoted by R_t – represents the **outcome**
- **ATE** = **difference in average outcome** between the **new** and **old** policy

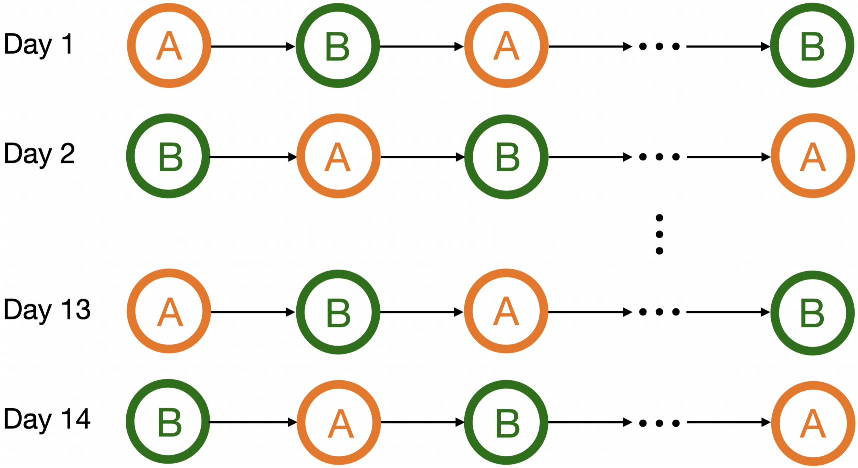
$$\lim_{T\to\infty}\left[\frac{1}{T}\sum_{t=1}^{T}\mathbb{E}R_t\right]-\lim_{T\to\infty}\left[\frac{1}{T}\sum_{t=1}^{T}\mathbb{E}R_t\right].$$

Letting $T \to \infty$ simplifies the analysis.

Alternating-day (AD) Design



Alternating-time (AT) Design



AD v.s. AT

Pros of **AD design**:

- Within each day, it is **on-policy** and avoids **distributional shift**, as opposed to **off-policy** designs (e.g., AT)
- On-policy designs are proven **optimal** in **fully observable Markovian** environments (Li et al., 2023).

Pros of **AT design**:

- Widely employed in ridesharing companies like Lyft and Didi [Chamandy, 2016, Luo et al., 2024]
- According to my industrial collaborator, AT yields **less variable ATE estimators** than AD

A Thought Experiment

- A simple setting **without carryover effects**:

$$R_t = \beta_{-1}\mathbb{I}(U_t = -1) + \beta_1\mathbb{I}(U_t = 1) + e_t$$

- ATE equals $\beta_1 - \beta_{-1}$ and can be estimated by

$$\widehat{\mathrm{ATE}} = \frac{\sum_{t=1}^{T} R_t\mathbb{I}(U_t = 1)}{\sum_{t=1}^{T} \mathbb{I}(U_t = 1)} - \frac{\sum_{t=1}^{T} R_t\mathbb{I}(U_t = -1)}{\sum_{t=1}^{T} \mathbb{I}(U_t = -1)}$$

A Thought Experiment (Cont'd)

The ATE estimator's asymptotic MSE under AD and AT is proportional to

$$\lim_{t\to\infty} \frac{1}{t}\mathrm{Var}(e_1 + e_2 + e_3 + e_4 + \cdots + e_t) \quad \text{and} \quad \lim_{t\to\infty} \frac{1}{t}\mathrm{Var}(e_1 - e_2 + e_3 - e_4 + \cdots - e_t)$$

which depends on the residual correlation:

- With **uncorrelated residuals**, both designs yield **same** MSEs
- With **positively correlated residuals**:
  - **AD assigns the same treatment** within each day, under which ATE estimator's variance inflates due to **accumulation** of these residuals
  - **AT alternates treatments** for adjacent observations, effectively **negating** these residuals, leading to more efficient ATE estimation
- With **negatively correlated residuals**, AD generally outperforms AT

When Can AT Be More Efficient than AD

Key Condition: Residuals are positively correlated

- **Rule out full observablity** (Markovianity) where residuals are uncorrelated.
- Can only be met under **partial observability**.
- Suggest partial observability is more realistic, aligning with my collaborator's finding.
- **Often satisfied** in practice:

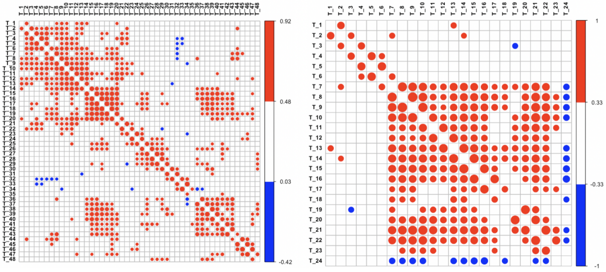


Figure: Estimated correlation coefficients between pairs of fitted outcome residuals from the two cities

Some Motivating Questions

- **Q1: Previous analysis excludes carryover effects. Can we extend the results to accommodate carryover effects?**

- **Q2: Previous analysis focuses on AD and AT. Can we consider more general designs?**

Our Contributions

- **Methodologically**, we propose:
  1. A **controlled (V)ARMA** model → allow **carryover effects** & **partial observability**
  2. Two **efficiency indicators** → compare commonly used designs (AD, AT)
  3. A **reinforcement learning** (RL) algorithm → compute the **optimal design**
- **Theoretically**, we:
  1. Establish **asymptotic MSEs** of ATE estimators → compare different designs
  2. Introduce **small signal condition** → simplify asymptotic analysis in sequential settings
  3. Prove the **optimal treatment allocation strategy** is ***q***-dependent → form the basis of our proposed RL algorithm
- **Empirically**, we demonstrate the advantages of our proposal using:
  1. A dispatch simulator (`https://github.com/callmespring/MDPOD`)
  2. Two real datasets from ridesharing companies.

Controlled VARMA Model

Consider a univariate controlled ARMA

$$Y_t = \mu + \underbrace{\sum_{j=1}^{p} a_j Y_{t-j}}_{\text{AR Part}} + \underbrace{bU_t}_{\text{Control}} + \varepsilon_t + \underbrace{\sum_{j=1}^{q} \theta_j \varepsilon_{t-j}}_{\text{MA Part}}$$

- **AR parameters** $\{a_j\}_j$ & **control parameter** b $\rightarrow$ **ATE**, equal to $2b/(1 - \sum_j a_j)$
  - Partial observability $\rightarrow$ standard OLS **fails** to consistently estimate b & $\{a_j\}_j$
  - Employ **Yule-Walker estimation** (method of moments) instead
  - Similar to **IV** estimation, utilize past observations as IVs
- **MA parameters** $\{\theta_j\}_j$ $\rightarrow$ **residual correlation** $\rightarrow$ **optimal design**

Theory: Small Signal Condition

- **Asymptotic framework**: large sample $\boldsymbol{T} \rightarrow \infty$ & small signal **ATE**$\rightarrow 0$
- **Empirical alignment**: size of ATE ranges from 0.5% to 2%
- **Theoretical simplification**: considerably simplifies the computation of ATE estimator's MSE in sequential settings. According to Taylor's expansion:

$$\widehat{\mathrm{ATE}} - \mathrm{ATE} = \frac{2\hat{b}}{1-\sum_j \hat{a}_j} - \frac{2b}{1-\sum_j a_j}$$

$$= \frac{2(\hat{b}-b)}{1-\sum_j a_j} + \frac{2b}{(1-\sum_j a_j)^2}\sum_j(\hat{a}_j - a_j) + o_p\left(\frac{1}{\sqrt{T}}\right)$$

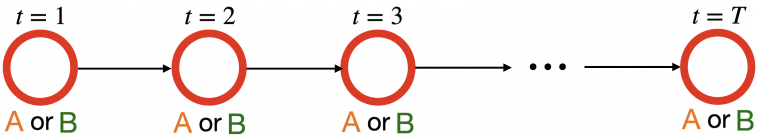
Leading term. Easy to calculate its asymptotic variance under weak signal

Challenging to obtain the closed form of its asymptotic variance, but negligible under weak signal condition

High-order reminder

Design

Identify **optimal design** that **minimizes MSE of ATE estimator**



We focus on the class of **observation-agnostic** designs:

- U_1 is randomly assigned
- The distribution of U_t depends on $(U_1, \cdots, U_{t-1})$, independent of $(Y_1, \cdots, Y_{t-1})$

It covers three commonly used designs:

1. Uniform random (UR) design: $\{U_t\}_t$ are uniformly independently generated
2. AD: $U_1 = U_2 = \cdots = U_D = -U_{D+1} = \cdots = -U_{2D} = U_{2D+1} = \cdots$
3. AT: $U_1 = -U_2 = U_3 = -U_4 = \cdots = (-1)^{T-1} U_T$

Design: Optimality

Theorem (Optimal Design)

The optimal design must satisfy $\lim_T \sum_{t=1}^{T}(\mathbb{E}U_t/T) = 0$. *Additionally, it must minimize*

$$\sum_{k=1}^{q}\left[\lim_{T}\left(\frac{1}{T}\sum_{t=1}^{T}\mathbb{E}U_t U_{t+k}\right)\underbrace{\sum_{j=k}^{q}\theta_j\theta_{j-k}}_{c_k}\right]$$

Objective: learn the optimal observation-agnostic design that:

(i) **Minimizes** the above criterion

(ii) **Maintains** a zero mean asymptotically, i.e., $\lim_T \sum_{t=1}^{T}(\mathbb{E}U_t/T) = 0$

Design: An RL Approach

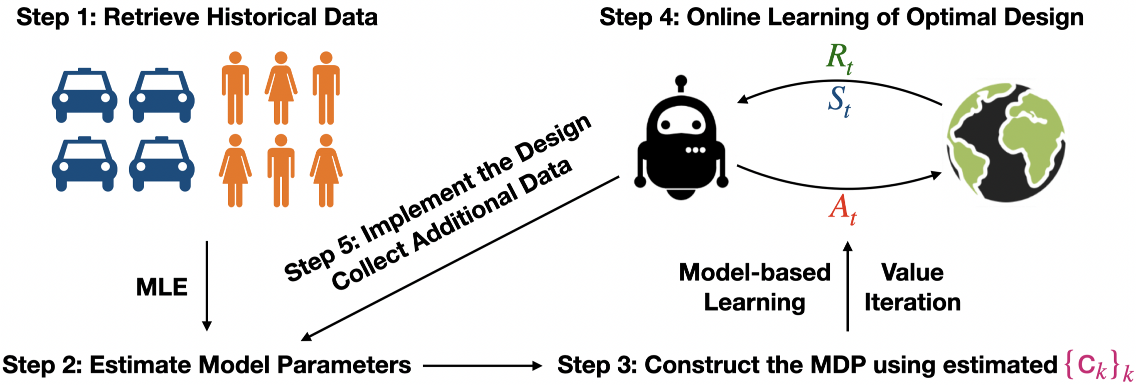
Solution: reformulate the minimization as an infinite-horizon average-reward RL problem

- **State S_t**: the collection of past q treatments $(U_{t-q}, U_{t-q+1}, \cdots, U_{t-1})$
- **Action A_t**: the current treatment $U_t \in \{-1, 1\}$
- **Reward R_t**: a deterministic function of state-action pair, $-\sum_{k=1}^{q} c_k(U_t U_{t-k})$

Easy to verify:

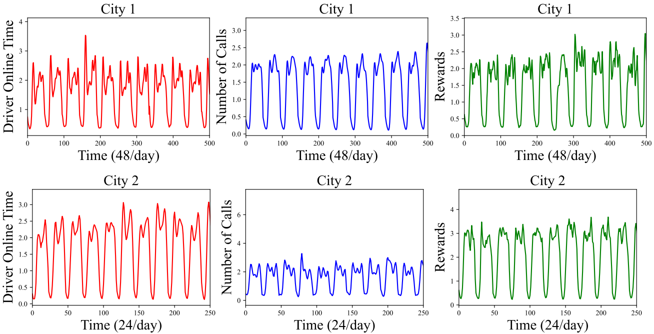
1. The minimization objective equals the negative average reward $\to$ equivalent to **maximizing the average reward**
2. The process is an **MDP** $\to$ there exists an optimal stationary policy maximizes the average reward $\to$ optimal design is q-**dependent**, i.e., U_t is a deterministic function of $(U_{t-q}, U_{t-q+1}, \cdots, U_{t-1})$ & this function is stationary in t
3. **Uniformly randomly** assign the first q treatments $\to$ the resulting design maintains a zero mean and is indeed optimal

Design: An RL Approach (Cont'd)



Empirical Study: Real Datasets

- **Data**:



- We incorporate a **seasonal** term in our controlled VARMA model to account for seasonality. Below are MSEs of ATE estimators under different designs

City	EI_1	EI_2	AD	UR	AT	Ours
City 1	20.98	-21.11	11.98	11.63	9.72	**8.24**
City 2	-4.89	0.22	9.64	30.04	546.79	**8.38**

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